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INTERDISCIPLINARY (MATHEMATICS, PHYSICS AND BIOLOGY) NATIONAL ASSESSMENT IN ROMANIA – PERSONALIZED TEACHING AND LEARNING PLANS –

Gabriela Streinu-Cercel[1], Bogdan Cristescu[2]

Abstract. With the increasing importance given to standardized assessment, in 2014, we implemented in Romania a national testing program using a unique test combining elements from mathematics, physics and biology, for 12-13 year-old students. We present the stages of implementing this testing. We defined six interdisciplinary competencies and we used a three-parameter logistic model to describe the link between the students' performance in tests and their corresponding abilities in mathematics, physics and biology. The novelty of this strategy resides in the manner of interpreting test results. Instead of being classically graded, each test is evaluated based on clustered codes, providing individual feedback regarding each student (personalized learning plans) as well as clustered feedback at class, school, regional and national level (personalized teaching plans).

1. INTRODUCTION

Most assessment programs offer standardized information regarding students' knowledge and competencies, useful for comparing and organizing students' results in a hierarchical system. However, most programs are not capable of offering feedback at various steps during the learning and teaching process. Furthermore, each testing system is specific to one domain, for example mathematics, and cannot be used interdisciplinarily. To overcome these shortcomings, we have constructed an interdisciplinary tool that can be prospectively used to identify the areas where individual or collective adjustment would increase the effectiveness of the learning and teaching process.

We present this specific national assessment strategy applied in Romania (External National Assessment), designed to measure what the learners of the 6th grade (12-13 years old) are able to do with the content of mathematics, physics and biology already learnt and to what degree the learners interconnect those contents. To our knowledge, this is the first nationally-applied strategy for personalizing the teaching and learning process.

The results of this External National Assessment are communicated to all the involved teachers (mathematics, physics and biology), to all the students and their parents. These results are analyzed, interpreted and applied on national, school, class

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and individual level. Using a computer application, the results can be interpreted for each student.

Teachers may use this national test to identify the individual learning needs of students, to define learning objectives, to adopt teaching strategies and to plan learning activities in order to initiate appropriate remedial activities (if needed).

Due to the fact that this test permits both a diagnostic assessment and a formative one, every teacher can provide personalized teaching assistance and individualized learning programs by identifying particular student's skills, attitudes and aptitudes, by modeling the student's abilities and making early decisions about the future learning opportunities for each student.

2. TEST ELABORATION STEPS

The test was elaborated in several steps: devising the interdisciplinary competencies, projecting the specifications matrix, designing the test, elaborating the specific items and the encoding pattern. The test was designed in accordance with the quality cycle: plan-do-check-act (Figure 1).

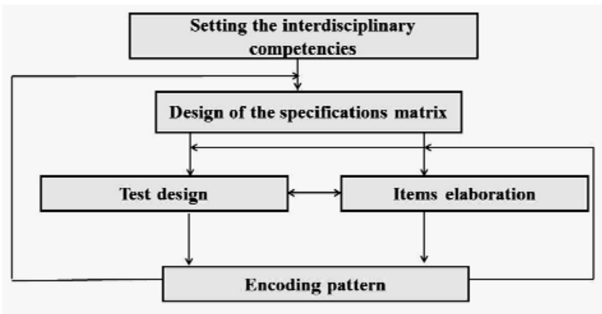

Figure 1: Steps of test elaboration

We started by devising the six interdisciplinary competencies (IC) that are derived from the general competencies and the specific competencies of the school curricula [1]. These six interdisciplinary competencies (IC) that we asses are**:**

IC1. **Identifying** data, concepts, specific relations of mathematics/science in an interdisciplinary context

IC2. **Processing** the following types of data: quantitative, qualitative, specific structural mathematics/science contained in various data sources

IC3. **Using** concepts, algorithms and procedures of mathematics/science to locally or globally characterize a particular case

IC4. **Expressing** the quantitative or qualitative characteristics of a particular situation in the specific language of mathematics/science

IC5. **Analyzing** the characteristics of relationships, phenomena or processes specific to mathematics/science, based on real or hypothetical situations

IC6. **Interpretation** of problem-situations specific to mathematics/science by integrating knowledge from different fields.

These competencies are stated in action terms and in accordance with Bloom's cognitive levels [2], and they are meant to link the stages of the teaching-learning-assessing process, the categories of skills, the informational feedback at skills level and the cognitive levels (Table 1).

Table 1: Correspondences among stages of teaching-learning-assessing process, skills, informational feedback and cognitive levels

	Stages of the teaching-learning-assessing process	**Categories of skills**	**Informational feedback at skills level**	**Cognitive levels**
IC1.	perception	reception	data input	knowledge
IC2.	internalization	initial processing (of data)	initial processing of data	understanding
IC3.	building mental structures	algorithmic thinking	primary processed data output	application
IC4.	transposition in language	expression	entry of primary processed data	analysis
IC5.	internal accommodation	secondary processing (of the results)	secondary processing of data	synthesis
IC6.	external adaptation	transfer	secondary processed data output	evaluation

As they are derived from the general competencies from the school's curricula in mathematics, the cognitive levels from the interdisciplinary competencies are the same with the ones in the school curricula in mathematics. The general architecture of the mathematics curricula in Romania is based on this principle (six competencies organized on the Bloom's cognitive levels, with input axis, processing axis and exit axis – Figure 2) and it is the same both for the general competencies and for the assessment competencies (for all national evaluations).

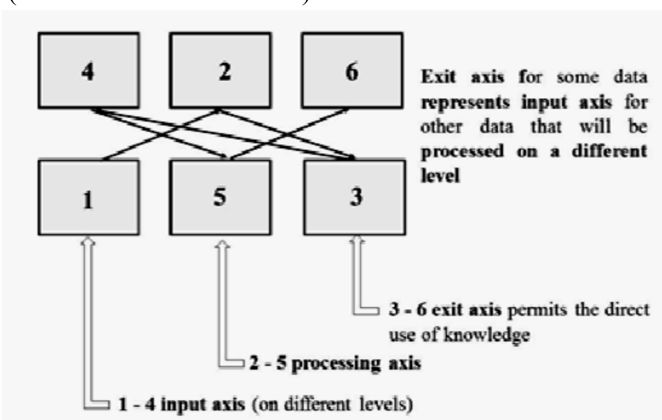

Figure 2: Informational feedback at skills level

The specification matrix is the instrument that certifies that the test measures the skills it aims to evaluate and that the test has content validity. The lines of the matrix state the content addressed and the columns of the matrix contain the levels of cognitive skills assessed.

The test contains 15 items of mathematics, physics and biology that are introduced in a general context and three specific contexts, as shown in Figure 3.

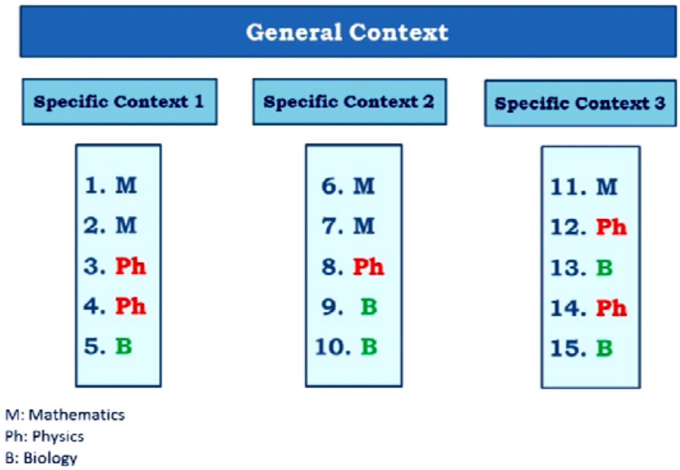

Figure 3: Test design

As shown, five of the items assess competencies acquired/developed mainly in mathematics classes, five of the items assess competencies acquired/developed mainly in physics classes and five of the items assess competencies acquired/developed mainly in biology classes. The items are not specific to one of the disciplines, for instance an item of physics can require units of measurement, transformations and calculus specific to mathematics.

We present examples of items used in the test and the encoding pattern used in order to evaluate the responses and to elaborate the individual feedback. Each test is evaluated based on clustered codes: total score, partial score and zero score.

Example 1 (*Item no. 11*, Test no.1) Code 21 11 12 13 00 01 99

One diorama exhibits 60 birds, mammals and insects. The number of birds represents 30% of the number of exhibits, and the number of mammals is equal to the number of insects. Determine the number of insects presented in this diorama.

Total score

Code 21: complete and correct reasoning and solving. Correct answer: 21 insects

Examples:

- $\frac{30}{100} \cdot 60 = 18$ birds

  $60 - 18 = 42$ and $\frac{1}{2} \cdot 42 = 21$, thus there are 21 insects
- $\frac{1}{2} \cdot (60 - \frac{30}{100} \cdot 60) = 21$ insects

- $100\% - 30\% = 70\%$, $\frac{1}{2} \cdot 70\% = 35\%$, that means that there are $\frac{35}{100} \cdot 60 = 21$ insects

  etc.

Partial score

Code 11: partially correct reasoning, calculations correct but incomplete

Example:

- $\frac{30}{100} \cdot 60 = 18$ birds, $60 - 18 = 42$

Code 12: partially correct reasoning, calculation errors

Example:

- $\frac{1}{2} \cdot (60 - \frac{30}{100} \cdot 60) = \frac{1}{2} \cdot 60 - \frac{30}{100} \cdot 60 = 12$ insects

Code 13: correct answer without justification. 21 insects

Zero score

Code 00: incomplete reasoning (correct statements) but not specific enough

Example:

- We can calculate the number of insects by subtracting the number of mammals and birds from the total number of exhibits

Code 01: other responses

Code 99: no answer

Example 2 (*Item no. 4*, Test no. 1)

Code	21	11	12	13	00	01	99

The ostrich is an excellent runner and can reach a speed of 70 km/h. Calculate the distance crossed by an ostrich running at this speed for 30 seconds. Express the result in meters.

Total score

Code 21: complete and correct reasoning and solving

Partial score

Code 11: correct reasoning, correct expression in appropriate units of physical quantities, calculation errors or incomplete calculations

Code 12: correct reasoning, inappropriate use of units of measurement

Code 13: correct answer without justification

Zero score

Code 00: wrong reasoning, correct expression in appropriate units of physical quantities

Code 01: other responses

Code 99: no answer

Example 3 (*Item no. 10*, Test no.1)

Code	2	1	0	9

Amphibians are a class of vertebrates, which, about 350-400 million years ago, left the waters to conquer land. Write down an adaptation of the amphibians to aquatic environment and one adaptation of the amphibians to terrestrial life.

Total score

Code 2: Specifying an adaptation of the amphibians to the aquatic environment and an adaptation of the amphibians to the terrestrial environment

Partial score

Code 1: Specifying an adaptation of the amphibians to the aquatic environment or an adaptation of the amphibians to the terrestrial environment

Zero score

Code 0: other responses

Code 9: no answer

The encoding pattern used to assess the items was created with the purpose of obtaining a valid feedback in order to be used in the future teaching-learning-assessing process. It is able to provide feedback for each competency acquired/developed as it is stated in the specification matrix and for every student that responded to the test.

3. Test Calibration

The *Item Response Theory* (IRT) [3] adopts explicit models for the probability of each possible response to each item and derives the probability of each possible response as a function of ability and some item parameters. IRT is the model we used to obtain the likelihood of ability as a function of the actually observed responses and the item parameters. The ability value that has the highest likelihood becomes the estimated ability.

The IRT model has to be true (correct) and the item parameters known. Two calibration studies have been performed (in 2012 and 2013) and the items were given to a representative number of tested students. Their responses were used to estimate the item parameters.

We used the *Three Parameter Logistic* (3PL) [4] model to describe the link between the students' performance in tests and their corresponding abilities in mathematics, physics and biology.

In the 3PL model each item is characterized by parameters a_i, b_i and c_i and the probability [4] of a random student having the ability θ to respond correctly to item i is given by:

$$P_i(\theta, a_i, b_i, c_i) = c_i + (1 - c_i)\frac{e^{a_i(\theta - b_i)}}{1 + e^{a_i(\theta - b_i)}},$$

where a_i is the item discrimination parameter, b_i is the item difficulty parameter and c_i is the probability of a correct response when true ability approaches $-\infty$.

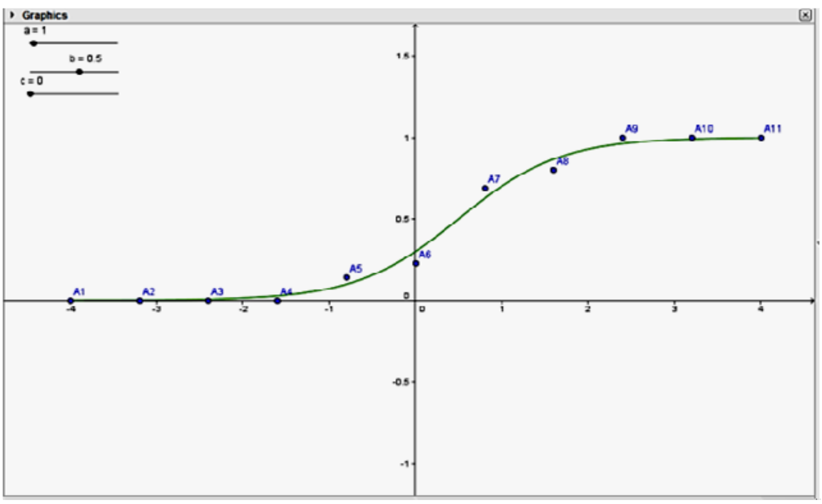

Figure 4: Characteristic curve for *Item no. 2*

The items were tested on equivalent samples (the samples for the tests are statistically identical – up to negligible random variation). The probabilities of correct answer for each item were calculated based on data collected from samples and from an initial estimate of the ability level. The ability level was initially estimated based on the score assigned to each student using their responses, the encoding pattern and the matrix of specifications.

The probability of correct answer was plotted depending on ability and we determined parameter values for each item. For example, in Figure 4 and Figure 5, we present the characteristic curve for two of the items from the test.

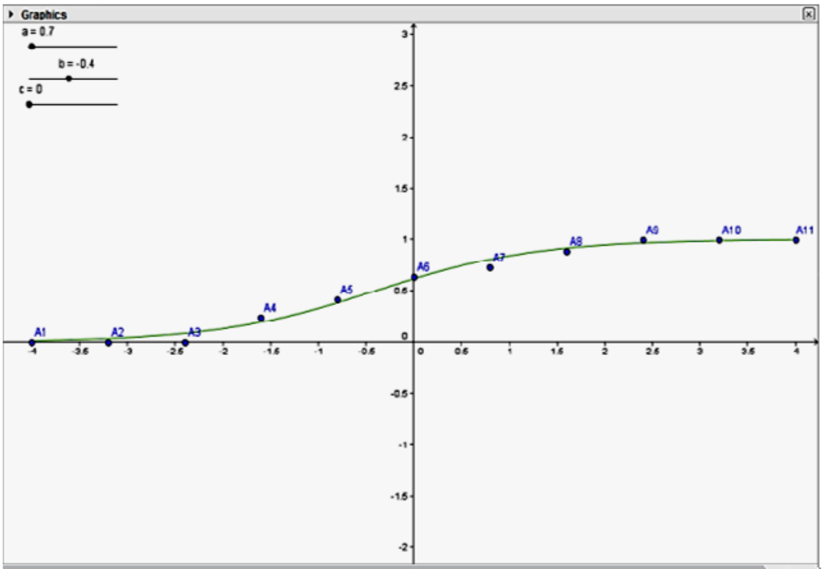

Figure 5: Characteristic curve for *Item no. 11*

The values thus determined for the parameters of the items have been used to calculate, separately for each test, the likelihood function:

$$L(\theta)=\prod_{i=1}^{n} P_i^{u_i}(\theta,a_i,b_i,c_i)(1-P_i(\theta,a_i,b_i,c_i))^{1-u_i},$$

where $u_i \in [0,1]$ is the score on item i; $u_i = 0$ if the student answered incorrectly at item i and $u_i = 1$ if the student answered correctly at item i. The ability of each student in the sample was fixed at the value that maximizes the likelihood function.

We take $\theta^* = l\theta + k$, where l and k are constants, i.e. we obtain a normal distribution of abilities for the students (mean of 0 and standard deviation of 1). We can adjust the 3PL model to accommodate the linear transformation of ability by taking $a_i^* = \frac{a_i}{l}$, $b_i^* = lb_i + k$ and $c_i^* = c_i$. Since $P_i(\theta^*) = P_i(\theta)$, the probability of a correct response is invariant to these transformations. For our test, the difficulty parameter b_i^* varied between -1.8 and $+1.3$.

The test characteristic curve was obtained for each test by plotting the probability of a student with the ability θ to obtain a certain score to the test, using $\xi(\theta) = \sum_{i=1}^{n} P_i(\theta)$.

The tests were constructed so that they have practically identical curves, independent of item-specific context.

4. INTERPRETATION OF RESULTS

The novelty of this strategy resides in the manner of interpreting test results. Instead of being graded, each test is evaluated based on clustered codes, providing individual feedback regarding each student (e.g., the curricular area where further work is needed – translated into personalized learning plans) as well as clustered feedback at class, school, regional and national level, identifying the areas where adjustment would increase the effectiveness of the teaching process – generating personalized teaching plans.

To each score presented in the encoding pattern we associated a description, followed by detailed examples in order to ensure that the team of teachers that evaluated each test will select the most appropriate code for each item and each student.

Furthermore, the encoding pattern represents the basis for a computer application that can extract for each student strong points and weak points, so that every teacher has the building bricks for designing an individualized learning plan for each student.

Each student also receives a list of strong points and weak points in relation with each of the six interdisciplinary competencies, but expressed in action verbs, and detailed facts for each content assessed.

In Table 2 we present the encoding pattern for Example 1 (*Item no. 11*), with the correspondences among the codes associated to the item, the descriptors for each code and the positive/weak remarks that are used to complete the personalized teaching plans.

Table 2: Presentation of encoding pattern for *Item no. 11*: codes, descriptors, positive remarks, weak remarks

Code	Descriptor	Positive remarks	Weak remarks
21	- complete and correct reasoning and solving	- correct usage of percentage, ratio or proportion	
11	- partially correct reasoning, calculations are correct but incomplete	- partially correct reasoning - correct usage of percentage, ratio or proportion but incomplete calculations	- the usage of percentage, ratio or proportion wasn't accurate enough in order to solve the problem
12	- partially correct reasoning, calculation errors	- partially correct reasoning	- calculation errors

13	- correct answer without justification		- didn't present any steps in reasoning or calculation leading to the correct result
00	- incomplete reasoning (correct statements) but not specific enough	- correct general statements about percentage, ratio or proportion, but not specific enough for this problem	- the correct statements written weren't applied to the actual data of the problem
01	- other responses		- didn't write any correct relations between the problem's data
99	- no answer		- didn't show any attempts to solve the problem

Using the test characteristic curve and the distribution of scores obtained, we can also determine the distribution of students among the school population, based on their ability in mathematics and science.

5. CONCLUSION

In conclusion, we have developed an interdisciplinary tool that can be prospectively used to provide individual feedback for each student pointing to the curricular area where further work is needed, as well as clustered feedback at class, school, regional and national level, highlighting the areas where adjustment would increase the effectiveness of the teaching process. The resulting *personalized learning and teaching plans* are generated through a novel evaluation process that applies clustered codes instead of classical grading systems. To our knowledge, this is the first nationally-applied strategy for customizing the teaching and learning process.

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[1)]National Assessment and Evaluation Center, Bucharest, Romania
E-mail address: gabriela.streinucercel@rocnee.eu
[2)] National Assessment and Evaluation Center, Bucharest, Romania
E-mail address: bogdan.cristescu@rocnee.eu .

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VISUALIZATION OF DISCRETE RANDOM VARIABLES

Zoran Trifunov[1], Elena Karamazova[2]

Abstract. In this paper, by use of information technology and existing definitions, a discrete random variable will be introduced, with emphasis on variables modeling probability situations with only two outcomes. The event can be repeated finite or infinite number of times. By analyses of examples, the Bernoulli, binomial and geometric random variables will be applied. Examples of discrete random variable with a geometric distribution will be given, which can be represented visually by using GeoGebra. The given problem will be visually presented by an applet developed in GeoGebra that will make the visual representation of the problem, i.e. the areas of definition and the favorable events easier. Then it will be solved mathematically.

1. INTRODUCTION

There are two types of random variables: random variables whose set of values is discrete (discontinuous or infinite) which for any $x_i \in R_X$, $P\{X = x_i\} \neq 0$, and random variables whose values set is an interval of real numbers, where $P\{X = x\} = 0, \forall x \in \mathbb{R}$.

The random variable X is of discrete type, if there is a discrete set $R_X \subseteq \mathbb{R}$ such that $P\{X \in R_X\} = 1$.

Example 1. Number of defective products under control of production, number of traffic accidents at a certain time, etc.

Random variables of discrete type usually are given with set of possible values $R_X = \{x_1, x_2, \ldots\}$ and sequence of real numbers $p_1, p_2, \ldots,\ 0 < p_i < 1, \sum_i p_i = 1,$ such that $P\{X = x_i\} = p_i$. Sets R_X and sequence numbers $p_i, i = 1,2,\ldots,$ determine the law of probability distribution of the random variable X of the form $X : (\begin{smallmatrix} x_1 & x_2 & \cdots & x_n \\ p_1 & p_3 & \cdots & p_n \end{smallmatrix})$, $\sum_{i=1}^{n} p_i = 1$ where $x_1, x_2, \ldots, x_n$ might be values of the random variable X and $p_1, p_2, \ldots, p_n$ probability that random variable X has for $x_1, x_2, \ldots, x_n$ accordingly.

Generally, the distribution function of a random variable X with the set values $R_X = \{x_1, x_2, \ldots\}$ is determined in the following manner

$$F_X(x) = \sum_{i: x_i < x} P\{X = x_i\},\ x \in R_X,$$

Key words and phrases. discrete random variable, the law of distribution, the cumulative distribution function

where counting is done by all i whose values x_i are less than the fixed x. If X receives n different values arranged by size $x_1, x_2, \ldots, x_n$ then

$$F_X(x) = P\{X < x\} = \sum_{x_i < x} p_i \quad ,$$

i.e.

$$F_X(x) = \begin{cases} 0, & x < x_1, \\ p_1, & x_1 \le x < x_2, \\ p_1 + p_2, & x_2 \le x < x_3, \\ p_1 + \ldots + p_{n-1}, & x_{n-1} \le x < x_n, \\ 1, & x \ge x_n. \end{cases}$$

2. VISUAL REPRESENTATION OF BINOMIAL, BERNOULLI AND GEOMETRIC DISTRIBUTION IN GEOGEBRA

Random variable X has a binomial distribution with parameters p and n, if the set of values X, $R_X = \{0, 1, 2, \ldots, n\}$ and

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k \in R_X. \tag{1}$$

Write $X \sim B(n, p)$ where n is the number of independent experiments, and probability p to a realization of an event. Example of visually presented random variable with binomial distribution for $p = 0,47, n = 90,$ using the software GeoGebra have in figure1.

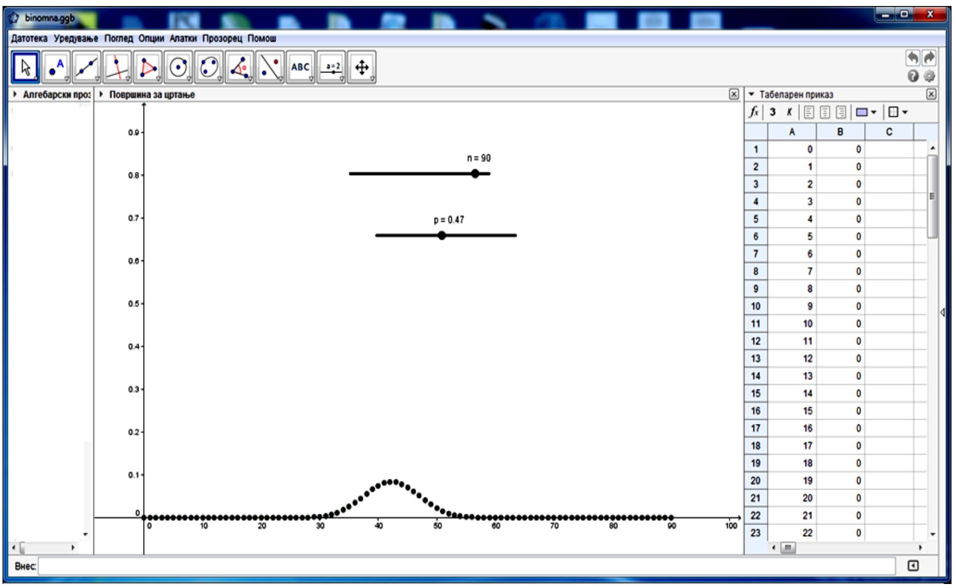

Figure 1

For $n=1$ the distribution $B(1,p)$ is called Bernoulli distribution with a probability of realization of an event p and $1-p$ if the event is not realized. Example of visually presented Bernoulli random variable for $p=0,4$ in software GeoGebra in figure 2:

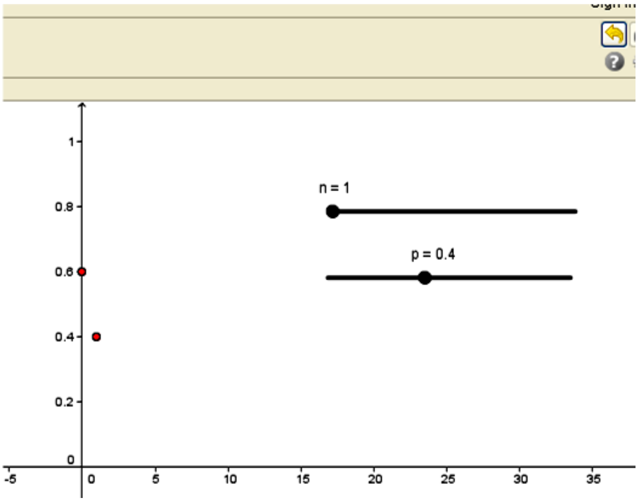

Figure 2

Random variable X has a geometric distribution with parameter p if

$$P\{X=k\}=(1-p)^{k-1}p, \quad k=1,2... \quad . \qquad (2)$$

An example of a random variable with geometric distribution for $p=0,3$, in software GeoGebra presented in figure 3:

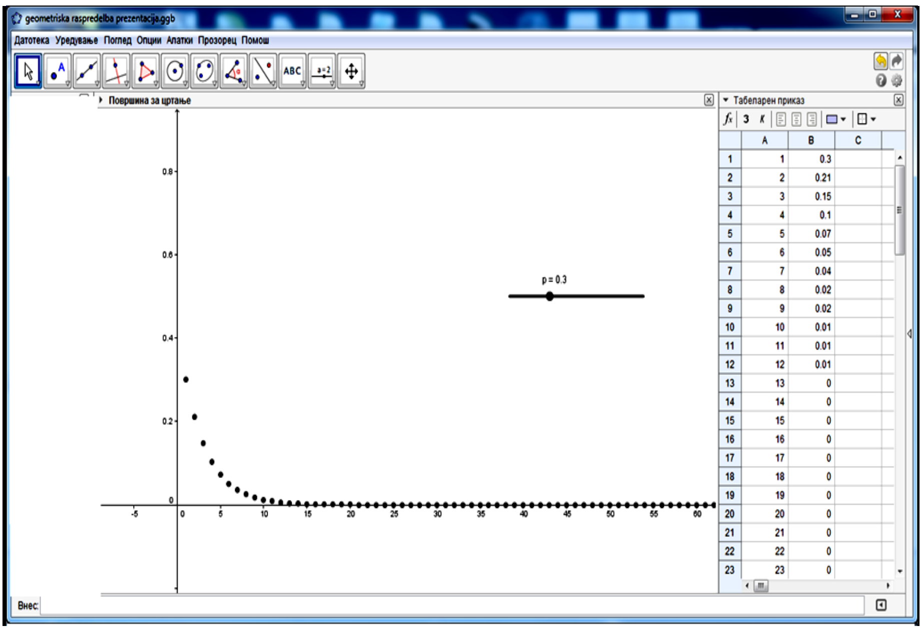

Figure 3

Figure 4 presents an applet for visualizing geometric and binomial distribution made in GeoGebra by which we can see how the change of n and p affect distributions. We use two spaces for drawing and two sliders. The first window represents a random variable with binomial distribution, and the second window a random variable with geometric distribution. One of the slides denoted by p, has a value of 0 to 1, step 0.01 which is the probability that the event A will occur. The second slider denoted by n has a value of 0 to 100, step 1 which is a number of repetitions of favorable events in 100 repetitions of the experiment. The slides are connected to the two windows and two graphs of distributions. By changing the value of p we visually represent the change of distributions and their graphical representation.

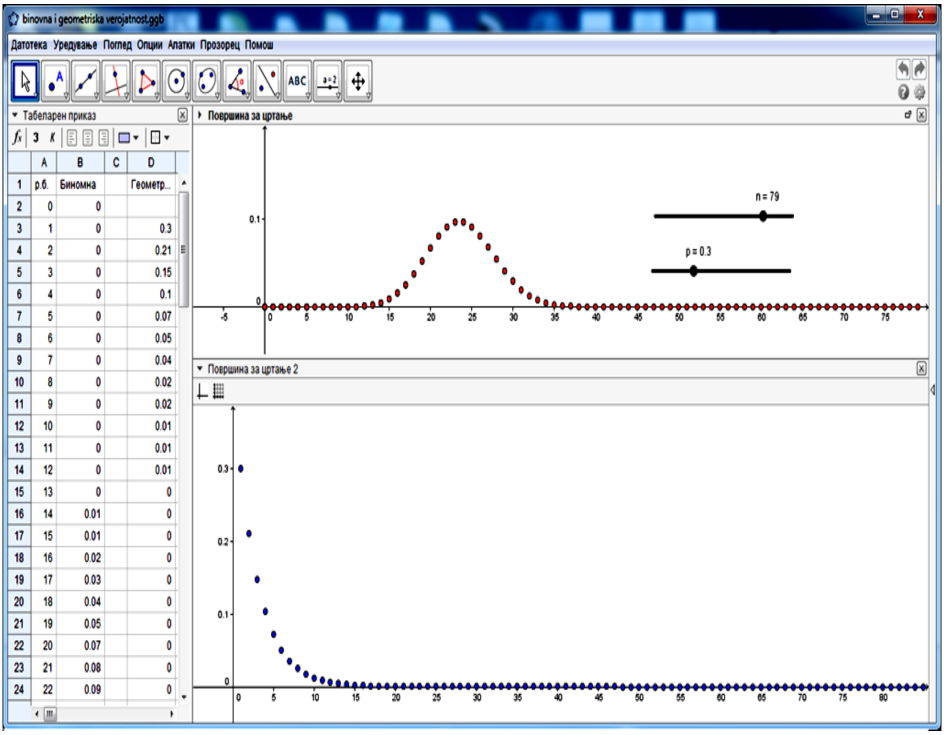

Figure 4

The following will give examples of discrete random variables of the same experiment that vary according to the number of repetitions of the experiment. Let the probability to hit the target in the shooting of a shooter be 0.7. Let

a) Y : be the number of target hits in one shooting

b) X : be the number of target hits, in 5 shootings,

c) Z : be the number of the shooting when the target is hit, if the shootings continue until the target is hit.

We will determine the type of the random variables, their laws of probability distributions and the cumulative distribution function. Using the previous applet we could visually present the law of distribution for the random variable. Let A be an event target hit.

$$p = P(A) = 0,7,\ P(\overline{A}) = 1 - P(A) = 1 - 0,7 = 0,3.$$

Y : be the number of target hits in one shooting. The random variable Y has a Bernoulli distribution.

$$R_Y = \{0,1\},\ P\{Y=0\} = P(\overline{A}) = 0,3,\ P\{Y=1\} = \mathrm{P}(A) = 0,7\,.$$

So the law of probability distributions PDF of the random variable Y is given in figure 5.

Y	P(Y)
0	0,3
1	0,7

Figure 5

The law of probability distributions PDF of the random variable Y presented in GeoGebra is given in figure 6.

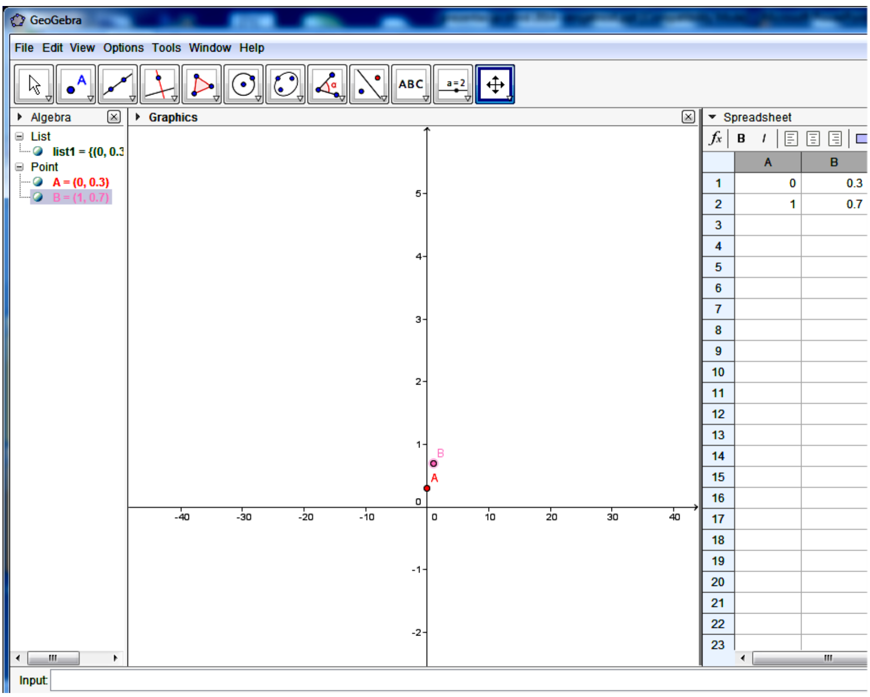

Figure 6

For the cumulative distribution function CDF of the random variable Y we have:

$$F(y)=\begin{cases}0, & y<0,\\ 0,3, & 0,3\le y<1,\\ 1, & y\ge 1.\end{cases}$$

The cumulative distribution function CDF of the random variable Y presented in GeoGebra is shown in figure 7:

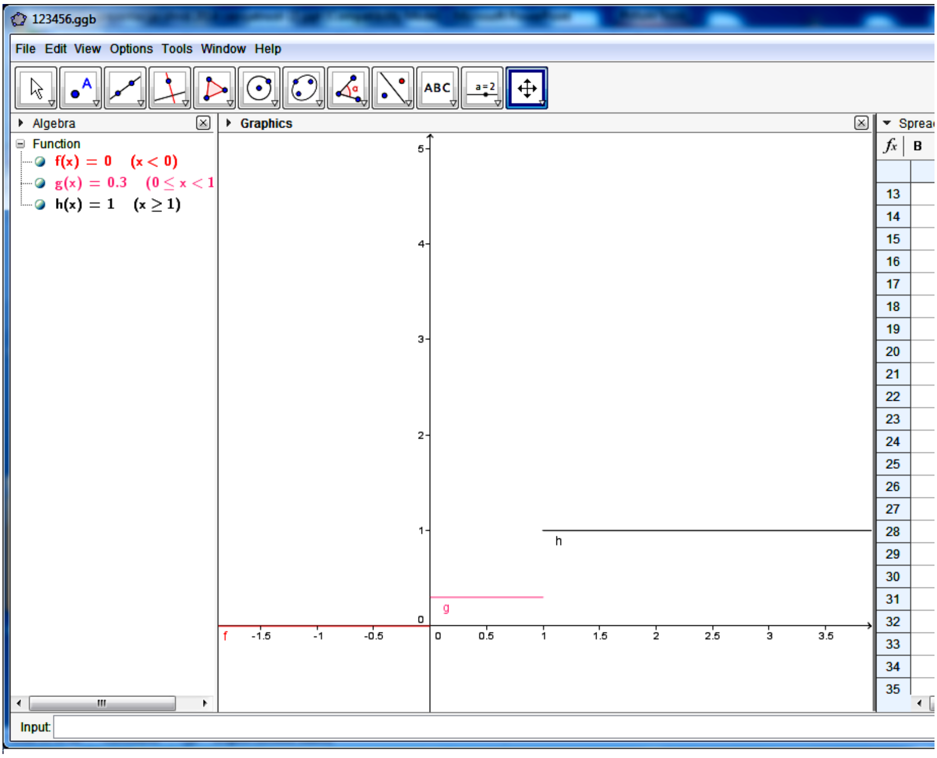

Figure 7

b) X is the random varible the number of target hits, in 5 shootings. The random variable X has a binomial distribution.

$$R_X=\{0,1,2,3,4,5\},$$

$$P\{X=0\}=\binom{5}{0}0,7^0\cdot 0,3^5=1\cdot 0,3^5=0,3^5=0,00243$$

$$P\{X=1\}=\binom{5}{1}0,7^1\cdot 0,3^4=5\cdot 0,7\cdot 0,3^4=0,02835$$

$$P\{X=2\}=\binom{5}{2}0,7^2\cdot 0,3^3=10\cdot 0,49\cdot 0,027=0,1323$$

$$P\{X=3\}=\binom{5}{3}0,7^3\cdot 0,3^2=10\cdot 0,343\cdot 0,09=0,3087$$

$$P\{X=4\}=\binom{5}{4}0,7^4\cdot 0,3^1=5\cdot 0,2401\cdot 0,3=0,36015$$

$$P\{X=5\}=\binom{5}{5}0,7^5\cdot 0,3^0=1\cdot 0,16807\cdot 1=0,16807$$

Therefore, the law of probability distributions PDF of the random variable X is given in figure 8.

X	$P(X)$
0	0,00243
1	0,02835
2	0,1323
3	0,3087
4	0,36015
5	0,16807

Figure 8

The law of probability distributions PDF of the random variable X presented in GeoGebra is given in figure 9.

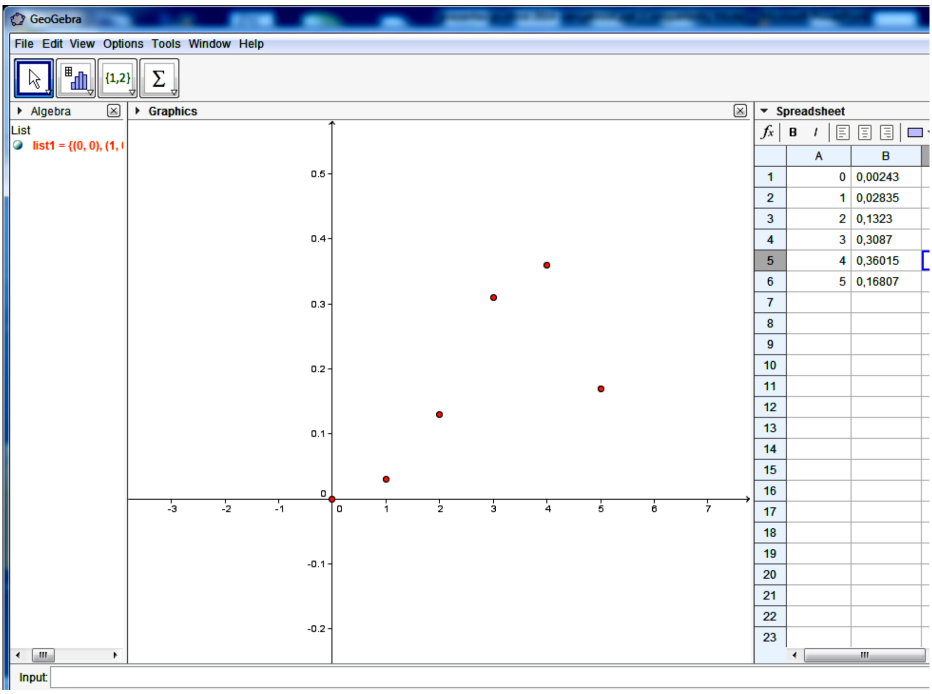

Figure 9

The cumulative distribution function CDF of the random variable X is:

$$F(x)=\begin{cases}0 & x<0\\ 0,00243 & 0\le x<1\\ 0,03078 & 1\le x<2\\ 0,16308 & 2\le x<3.\\ 0,47178 & 3\le x<4\\ 0,83193 & 4\le x<5\\ 1 & 5\le x\end{cases}$$

The cumulative distribution function CDF of the X presented in GeoGebra is given in figure 10.

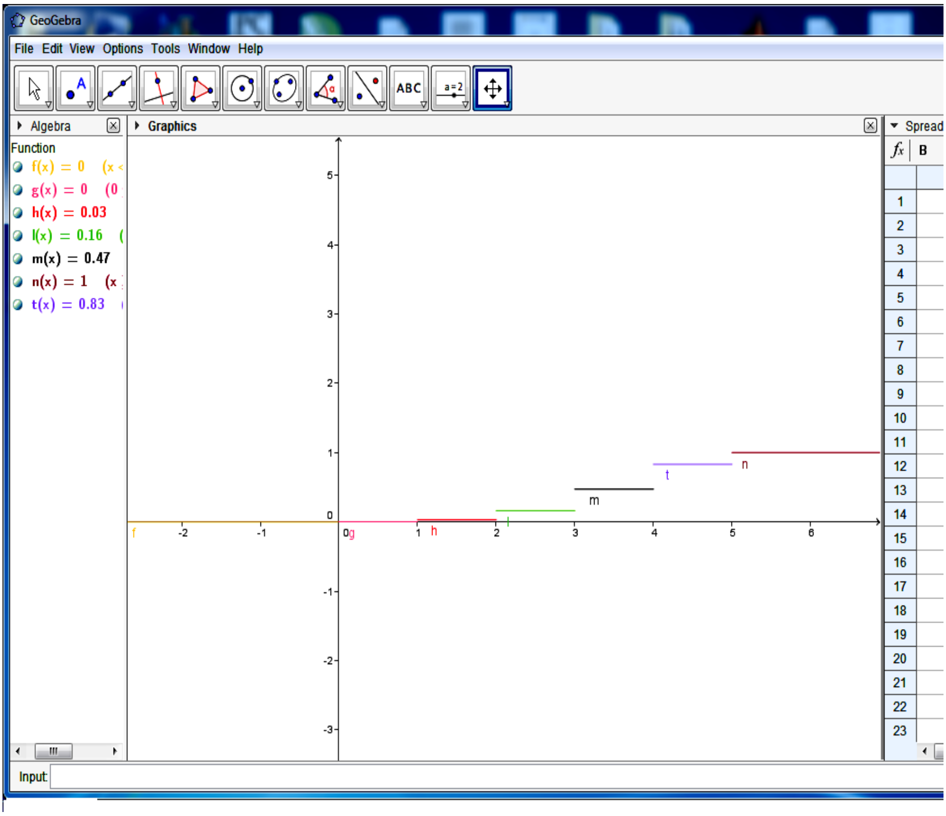

Figure 10

c) Z is the number of the shooting when the target is hit, if the shootings continue until the target is hit. The random variable Z has a geometric distribution. The set of values is

$1-$ target hit at the first attempt

$2-$ target hit at the second attempt

....

$n-$ target hit at the n th attempt

……………………………………

$P\{Z=1\}=P(A)=0,7$,

$P\{Z=2\}=P(\overline{A}A)=0,3\cdot 0,7=0,21$,

$P\{Z=3\}=P(\overline{A}\overline{A}A)=0,3^2\cdot 0,7=0,063$,

………………………………………….

$P\{Z=n\}=P(\underbrace{\overline{A}\overline{A}...\overline{A}}_{n-1}A)=0,3^{n-1}\cdot 0,7$,

………………………………………..

$$\begin{aligned} P\{Z=1\}+P\{Z=2\}+...+P\{Z=n\}+... &= 0,7+0,3\cdot 0,7+0,3^2\cdot 0,7+...+0,3^{n-1}\cdot 0,7+... \\ &= 0,7(1+0,3+0,3^2+...+0,3^{n-1}+...) \\ &= 0,7\cdot\frac{1}{1-0,3}=1. \end{aligned}$$

3. CONCLUSION

This paper visually represents discrete random variables with binomial, Bernoulli and geometric distribution through the software GeoGebra. With the visual representation of random variables we want to achieve our goal of students easily learning and recognizing random variables in solving tasks.

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[1] *Goce Delcev University,*
email: zoran.trifunov@ugd.edu.mk
[2] *Goce Delcev University,*
email: elena.gelova@ugd.edu.mk