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CREATION OF THE BANK OF CONTEXTUAL TASKS WITH ECONOMIC CONTENT FOR THE FORMATION OF THE FINANCIAL LITERACY OF STUDENTS

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Abstract. At the moment the problem of the low level of financial literacy of the population is paid great attention to both in Russia and in other countries. One of the objectives of financial education is to teach children to use mathematical methods for making economic decisions. As one of the means of achieving this purpose we use contextual mathematical tasks. Our purpose was to create a bank of contextual mathematical tasks with economic content and a database for it. For further application of the received skills in real life, the students working with contextual tasks must learn to independently select the data they find important from a great variety of available data. Our purpose was to test the bank of tasks and its database that we had developed. In several testing groups of different ages we studied children's readiness to make a conscious choice of the necessary data for decision-making. One more purpose of the testing was to identify what data are important and what data are not taken into account when students solve a particular contextual task..

1. INTRODUCTION

Nowadays both in Russia and in a lot of developed and developing countries of the world the problem of increasing financial awareness of the population is paid much attention to. Solving this problem is considered to be one of the most important factors of developing economy of any country. A high level of financial literacy is crucial for achieving personal financial goals, organizing one's life, and, consequently, increasing standards of living and confidence in the future, which ultimately leads to the stability in the society and prosperity of economy. But these days many people make important financial decisions at random and without due consideration, only taking into account advice of their acquaintances, who are far from being financial experts. People often live from paycheck to paycheck and even oftener – beyond their means. They buy cars or expensive household appliances on credit without thinking about their ability of paying the money back and the risks connected with it. Statistical investigations conducted in Russia show that about a third of the Russians do not save money and do not know what taxes they pay. Only 33-37% of the Russians know something about the remote services, etc. In many countries of the world active efforts are made to introduce financial literacy as an independent subject or as part of the subjects currently studied in educational institutions. For example, in France personal finance knowledge is included in the course of mathematics. For instance, already in the third form children study the notion of income and learn to count taxes as based on the salary, etc.

In modern Russia some topics, necessary for forming financial literacy, are included in the courses of many school subjects. For example, the course of mathematics comprises tasks, aimed at forming the ability to use the acquired mathematical knowledge and skills in practical activities and in everyday life. Moreover, some schools realize the programmes of extracurricular activities in this sphere, for example, the programmes of the course «Aflatoun: child social and financial education». But students gain this knowledge in an unsystematic way. Moreover, in many cases the possibility to use the acquired knowledge in real life does not arise simultaneously with the acquisition of this knowledge at school. In the end all this does not yield the result, which could be obtained, if all the knowledge and skills were formed within a system. The main purpose of this study was to gather data about everyday situations that require mathematical calculations for decision-making; the age of students that face these situations; the selection criteria used by schoolchildren of different ages. Using the collected data, we will create a methodology of working with contextual mathematical tasks having economic content.

2. KEY STAKEHOLDERS

In 2011 the Ministry of Finance of the Russian Federation together with the World Bank initiated a pilot project aimed at increasing financial awareness of the population. In the initial stages the project was active only in two regions of the Russian Federation, but in 2014 it came to cover ten of them. One of the performers of the regional programme «Increasing financial awareness of the population and developing financial education in the Arkhangelsk region in 2014 – 2019» is Northern (Arctic) Federal University.

The project was launched. Teaching financial literacy is introduced into primary, secondary and higher education. However, the analysis of different sources shows that there are a lot of questions connected with the implementation of this teaching.

Question 1. Should schoolchildren be taught financial literacy in the classroom or during extracurricular activities?

Specialists of the Ministry of Finance and the Ministry of Education, teachers and parents have not yet agreed on how the new subject will be introduced into the educational standard and which subjects will have to share their academic hours with it. Moreover, the project was started on the initiative of the Ministry of Finance, but the problem of academic hours distribution is to be solved by the Ministry of Education.

Question 2. Who will conduct financial literacy classes?

The Ministry of Finance maintains that one of the main objectives of the project is to contribute to creating effective methods of teacher training and developing teaching culture in the sphere of personal finance. However, experts of the Ministry of Education of the Russian Federation are eager to know where to find teachers who have economic education and who are ready to give a small amount of lessons at school.

Question 3. What exactly will children be taught within the frames of the financial literacy course?

The views of the Ministry of Finance and parents on what exactly children should be taught as far as financial literacy is concerned are virtually identical.

For example, here is the opinion of one of the parents:

- we can persuade children that your bank deposit is bound to be constantly increasing whatever happens, but it would be better to explain to them that no one is immune from the crisis and the total loss of their money.

Almost the same is stated, for example, in the regional programme called «Increasing financial awareness of the population and developing financial education in the Arkhangelsk region in 2014 – 2019», in which the key performance indicators of the programme are:

- developing the ability to adequately assess the benefits and risks when choosing financial products;
- demonstrating the importance of creating a financial «safety cushion» against the possibility of emergency and crisis situations.

Here is the opinion of another parent:

- children will be made to follow the financial behavior strategies that serve the interests of financial structures.

One more quote from the same programme:

- developing the ability to be discriminating in the sphere of credit, deposit and insurance services...

Despite the similarity of views, parents object to the introduction of the financial literacy course into the school curriculum. They believe that economics and social science lessons are enough to make children feel confident in the world of finance. As a former economics schoolteacher, I know for sure that the current number of academic hours allocated to economics and social science in the curriculum is not enough even to prepare pupils for the Unified State Exam in social science.

So, the situation is quite strange. Specialists of the Ministry of Finance and parents of the children, who are going to be taught financial literacy, find it necessary that children acquire the same knowledge and skills. However, parents see no reason to incorporate special lessons into the curriculum, despite the fact that only a small percentage of children have the opportunity of developing the needed skills in the family. The problem of misunderstanding probably lies in the fact that children should be taught by teachers and not by financial experts.

3. REQUIREMENTS FOR CONTEXTUAL MATHEMATICAL TASKS

With an eye to solving this problem, since 2012 Russian scientists under the guidance of Professor Sergeyeva have been testing the programme «Aflatoun: child social and financial education», which makes up a part of an international educational project, whose secretariat is located in Amsterdam.

The implementation of this project removes a number of the abovementioned problems. First, the course is implemented as a part of extracurricular activities in the form of electives, clubs, and homerooms. Accordingly, the course does not reduce the teaching of the core subjects. Moreover, it is optional and is therefore taken only by those children whose parents want their child to study financial literacy. Secondly, due to the usage of the fully developed educational materials, the classes can be conducted by elementary school teachers as well as by economics, geography, mathematics, social science teachers, etc. Third, the basic concept of the course is a balanced approach to child social and financial education, which makes it possible to discuss issues of financial management in the context of solving social problems, not personal gain.

As the work on the scientific and methodological support of the international project «Aflatoun: child social and financial education» began, Russian teachers were given a task to analyze the educational materials suggested by the project's organizers. Having analyzed the materials, teachers came to the conclusion that the realization of the main purpose of the course, i.e. enabling students to make the most use of opportunities in life, is hindered by the lack of project materials connected with mathematical calculations. Thus, teachers of the Academy of public administration (Moscow), and Northern (Arctic) Federal University (Arkhangelsk), who have undertaken the scientific support of the project in Russia, faced the need to develop the mathematical content of the financial literacy course.

In our view, the purpose of teaching mathematics in general and within the financial literacy course in particular must be forming the ability of constructing mathematical models. As one of the means of achieving the desired goal we suggest contextual mathematical tasks.

By contextual mathematical tasks we understand:

- practice-oriented tasks,
- containing the description of a particular real-life situation,
- which is related to the student's individual experience
- and is personally significant for the student.

The mentioned criteria of contextual tasks made it necessary for us to develop these tasks in strict adherence to certain requirements.

The first requirement: «the situation which is presented in the task should be known to the student».

The first step in creating the bank of contextual tasks and the database for solving them was to select the situations that children face in real life and that require mathematical calculations to be optimally solved. Taking into consideration the financial activities faced by the child in everyday life, we identified types of contextual tasks which, in our opinion, should be used in the financial literacy course (Table 1).

Table 1: Types of contextual tasks

Financial literacy, socialization	Expenditures	Savings	Replenishment
Personal money	What are the best ways to spend the money? Do I have enough money? How many things can I buy? Purchases in Internet shops, computer games.	Estimation of the time required for saving money. How much money can I save by discount?	Shall I earn a lot? Calculation of net profit.
Consolidated personal finances	A fair division of joint purchases. Measuring the size of the debt/loan.	Estimation of the time of debt reimbursement. Estimation of the time required for joint savings.	Distribution of the joint income.
Household budget	Planning household budget expenditures. Purchases while travelling abroad. Choosing a payment method. Compilation of an expenditure report. Choosing a tariff or credit plan.	Economizing on utility costs. Tax deductions. Discount card purchases. Choosing a bank deposit. Measuring the size of bank savings.	Economizing on utility costs. Tax deductions. Discount card purchases. Choosing a bank deposit. Measuring the size of bank savings.

The second requirement: «solving the task must be personally significant for the student».

The next step in compiling the bank of contextual tasks is analyzing the age limits within which children have the opportunity of gaining personal experience in using financial resources. In addition to children’s interest in a particular kind of financial activity, these age limits should agree with the legal aspects of the use of financial resources. One of the examples is given in Table 2.

Table 2: Age limits and legal aspects of the use of financial resources

Types of financial activities	Directions of financial literacy/socialization	Age limits	Legal grounds
Replenishment of personal money	Calculation of net profit	From 14 years old	Civil Code of the Russian Federation in the part of the rights of underage children (Article 26)

The third requirement: «the task must have multiple ways of solution and criteria of «correctness».

The third requirement on contextual tasks led to the understanding that the problem situations of such tasks must be radically different from those that are presented in common school tasks. At least, they should contain redundant data, from which a child is supposed to select what he or she thinks is really needed.

Example 1. Mother sent her son Misha to the shop. She gave him 320 roubles and a list of products to buy: a loaf of wheat bread, half a loaf of rye bread, a package of milk, a stick of butter, 0.5 kg of sausages. Mother allowed Misha to spend the change on a bubble gum. There are two shops nearby which have the following prices. Which is the best shopping variant for Misha?

Table 3: Data to make a decision

Products	Prices in the shop «Dieta», roubles	Prices in shop «Sever», roubles
Wheat bread, a loaf	30	32
Sausages, 1 kg	244	212
Rye bread, a loaf	28	30
Milk, a package	75	72
Butter, a stick	78	84
Bubble gum, a piece	19	21

The fourth requirement: «the solution to the task must have an application area in everyday life».

This requirement shows that the child must be able to use the acquired skills not for solving standard tasks, but in real life. We need some means that enables the student to select from a variety of data those ones which, from his or her point of view, are significant in terms of «correctness» of solving the task.

By way of example, let us consider the situation suggested earlier: calculation of net income. Let us assume that a high school student has made up their mind to choose a summer job. On a classifieds site the student found a number of job offers. The offers contained the information on the vacancy, wages, company, employment schedule, and contacts (Table 4, Table 5).

Table 4: Data to make a decision

Date	Vacancy/salary (roubles)	Company
12:35 01.07.2014	Bijouterie maker Salary: 36000	Tsvetokamen Ltd.
13:25 30.06.2014	Typist (at home) Salary: 30000	Vizavit Ltd.
05:39 16.04.2014	Freelancer (home PC) during free time Salary: 15000 - 30000	SEO
02:12 24.03.2014	Typist (at home) Salary: 30000	Gilas Ltd.
07:44 15.03.2014	Group administrator Salary: 850	ordersmartework

Table 5: Data to make a decision

<table>
<tr><td>Sex: does not matter
Age: from 14 to 60
Education: does not matter
Experience: does not matter, without experience</td><td>City: Arkhangelsk
Company: Tsvetokamen Ltd.
Work schedule: flexible hours
Salary: 36000</td></tr>
<tr><td colspan="2">Additional information:
We are offering a normal home-based job of bijouterie maker (beads maker). This job is quite simple, does not require preliminary training and can be done by any pupil. For each 140 beads we offer 18000 roubles. Generally one man can make 140 beads within 25 days spending only one hour per day for this job. If you are interested in our offer, please visit our website http://cvetokamen.com for further details. All your questions are welcome by e-mail: cvetocamen@gmail.com
Tel.: http://cvetokamen.com
E-mail: cvetocamen@gmail.com
Contact person: Elena</td></tr>
</table>

The following questions arise: which job offer is the most advantageous one in terms of income? What additional information is required to make a decision on which job to take?

In our opinion, to make the economically right decision in terms of the amount of net income, the student should take into consideration the following additional information, which is not presented on the site.

1. The cost of travel to the place of work and back.
2. The possibility of having lunch at home or its cost in the nearest café or canteen.
3. The possible need for training and its cost.
4. The type of payment: piecework or hourly; the worker’s rights in case of hourly payment, etc.

We conducted the experiment in 3 test groups of different ages and studied children’s degree of readiness for making a conscious choice of the necessary data for making a decision. A total of 189 students from two schools aged from 9 to 17 have been tested. 90 students are from elementary school, 73 of them are from middle school, 26 students are from high school. For this purpose, the children were given a description of a real-life situation, which students of the particular age are familiar with, and it was suggested to them to make a decision that they thought was optimal in that situation. The assignment was the same for primary, secondary and high school children.

The task definition: «Your class has decided to hold a holiday and arrange a tea party afterwards. It was also decided that everybody should bring their own treat to the party. Moreover, the parents made up their minds to collect 100 roubles per person for the holiday party. A part of the collected money was allocated for buying half a liter of juice per person, and the remaining sum of money was supposed to be used for buying small gifts for contests. What juice would you recommend to buy?»

Primary school children were offered these tasks in a visual form. They were given samples of the offered products with their prices. Secondary school children were given printed materials. If the child thought that there were not enough data to make a decision, he or she was asked to explain what information was missing and why he or she thought so. In case of difficulties they could ask the teachers for help. Children could name several selection criteria.

Selection criteria used by the students of all ages (Diagram 1, Diagram 2, Diagram 3).

Diagram 1: Selection criteria of primary school children.

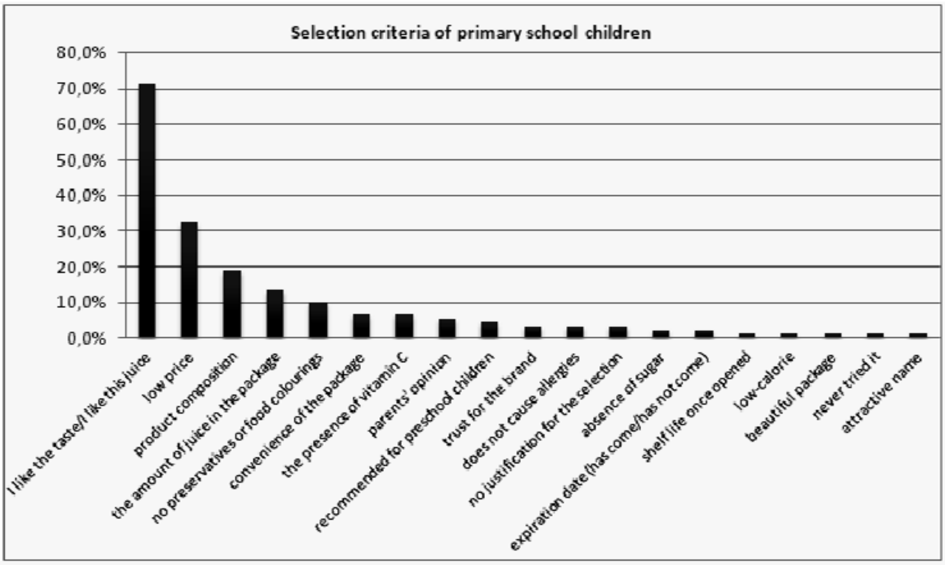


Diagram 2: Selection criteria of secondary school children

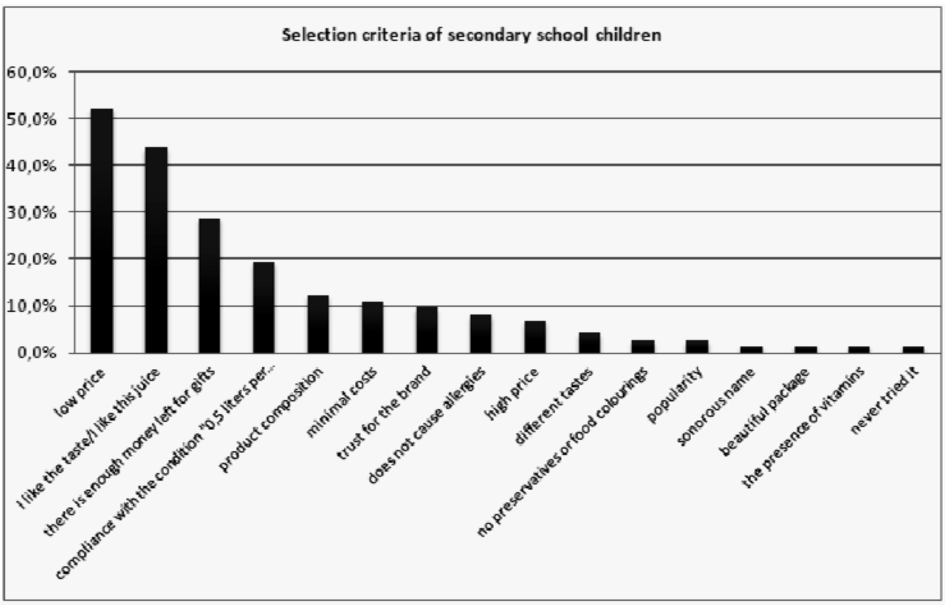
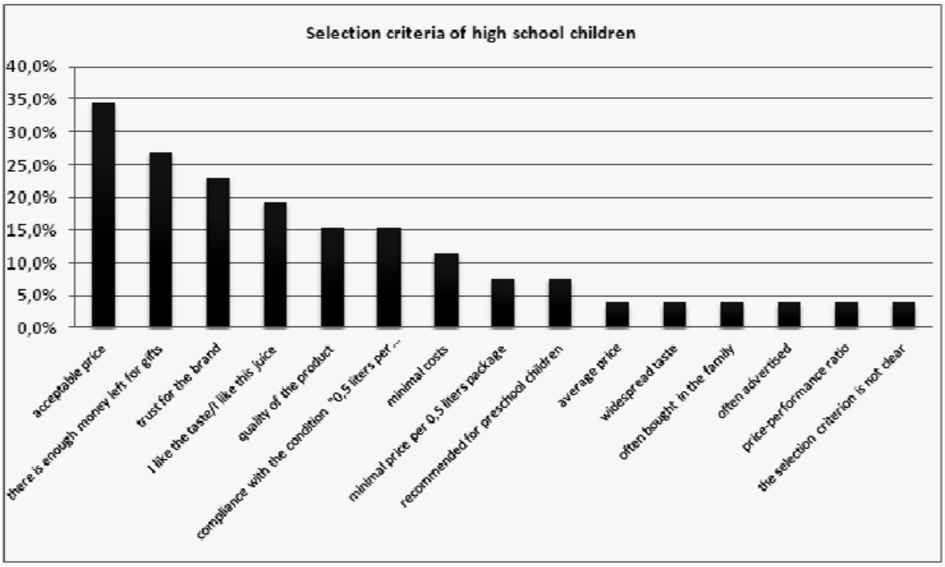


Diagram 3: Selection criteria of high school children



The results of the experiment. From 15 to 18 different selection criteria were used by the students. But for the vast majority of students the main criterion was the lowest price – 32 per cent of elementary, 52 per sent of secondary and 35 per sent of high school children. As the second important criterion they used their own opinion about the taste. This criterion was used by 71 per cent of elementary, 44 per sent of secondary and 19 per sent of high school children. Only students of primary school made their decisions basing on the real data specified on the product package. The middle and high school students, with the exception of one child, did not request any data for making a decision and used the initially suggested information about the price and quantity of product in one package. They relied exclusively on their own subjective opinions and often came to wrong conclusions. Only one of the students examined the quality-price ratio. It was an elementary school student. He noticed that in terms of the neat juice content the cheapest juice took almost the middle position among more expensive kinds of juice. Almost all schoolchildren examined only the product price without calculating the unit value.

The results of the experiment show that students of different ages are not ready to select the missing data which are necessary for making a well-thought-out and well-founded decision.

4. FINAL REMARKS

So, in our view, the definition of a contextual task should include:

- the description of the decision-making context;
- the necessary but not sufficient data for making a decision.

Except for the definition of the task, the students must be given access to additional information, which they find important for making a decision. It requires creating a database containing all the necessary information that can be requested by students. The information in this database should be up to date at all times in a particular region. It should also mention the access point to this information in a real situation. Creating the bank of tasks and the database is what we are doing at the moment.

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DEVELOPING COMMUNICATION COMPETENCES IN THE CONTEXT OF MATHEMATICS EDUCATION

Toni Chehlarova, Evgenia Sendova

One should not aim at being possible to understand, but at being impossible to misunderstand.
Quintilian, circa 100 AD

Abstract. The paper presents the experience of the authors related to the development of key competences in the STEAM education in the context of the European projects *KeyCoMath* and *KeyCoNet,* as well as within two gifted education programs in Bulgaria and USA. Resources of the *Virtual School Mathematics Laboratory* (*VirMathLab*) are discussed with emphasis on enhancing the communication competences in the mother tongue and foreign languages.

1. INTRODUCTION

Communication competence means that one is comfortable using a broad range of communication styles, and could choose appropriate and effective ways to communicate ideas to different audiences in diverse situations. The European Parliament and of the Council of the European Union gave in 2006 recommendations on key competences for lifelong learning [1]. This framework defines eight key competences and describes the essential knowledge, skills and attitudes related to each of them, viz: *communication in the mother tongue; communication in foreign languages; mathematical competence and basic competences in science and technology; digital competence; learning to learn; social and civic competences; sense of initiative and entrepreneurship; cultural awareness and expression.* These skills are necessary for all citizens for personal fulfilment, active citizenship, social inclusion and employability in a knowledge society. Forming key competences embraces the whole conscious life but it is the school education which is fundamental in it. Mathematics education plays a special role in this respect since it could provide appropriate conditions for developing each of the key competences. This is the basic idea behind the European project "Developing Key Competences through Mathematics Education" (KeyCoMath) [2]–[4] which aims at developing, implementing, and evaluating ways of working according to [1].

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Key words and phrases. competences, mathematics education, IBL, contests

2. Gaining communication competences in various settings

Further below we are considering some activities of the *Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences* (IMI-BAS) within this project related with the development of several key competences with an emphasis on the *Communication in the mother tongue*. This competence is determined by KeyCoMath [2] as one of integrating *doing mathematics and communicating with others orally or in written form. Pupils are encouraged to talk about mathematics, to discuss ideas, to write down thoughts and reflections and to present results.*

In addition, learning environments and beyond school forms will be presented as having proved their potential for developing also the key competences: *communication in foreign languages*, *mathematical competence and basic competences in science and technology*, *digital competence*, and *cultural awareness and expression*. Let us first throw a glance back at the roots of integrating the mathematics, informatics and language education as implemented in a Bulgarian context.

2.1. Language and mathematics — an integrated subject from 30 years ago

An innovative idea of integrating the study of mathematics, natural languages (Bulgarian, Russian and English) and a computer language (Logo) was launched in fifth grade of an educational experiment of the *Research Group on Education* carried out by the IMI-BAS jointly with the Ministry of Education which started the development of a new curriculum in 1979. Designed to show the intersection of language study with mathematical thinking in the context of informatics, the experimental textbook *Language and Mathematics* [5] included problems on translating from a natural to a formal language ordered by how difficult it was to understand the concepts and do the translation. This textbook contained also an algorithmic description of basic grammar rules and ways to extend the Logo turtle vocabulary in several languages. Applications of informatics notions (e.g. *cycle* and *recursion*) were shown in mathematics, physics, music, graphical design, so that every student could choose a problem according to his/her interest. Since computation provides new tools for self-expression, the students dared to explore areas they had previously considered inaccessible. The specially designed computer microworlds provided convenient tools for the students to deal with new notions from a procedural rather than from a declarative point of view, i.e. in the style of *how to* rather than *what is*. This has already had an impact on the way we started teaching mathematics emphasizing on its connections with literature, art, and music and conveyed the same spirit of inquiry based education (IBE) in our present courses for professional development of teachers.

2.2. EXPERIMENTS WITH CUBICAL CONSTRUCTIONS

An interesting experiment was designed and carried out by the authors within activities primarily aimed at the development and enhancement of the 3D imagination of students. This time the emphasis was on a verbal/written description of a construction of unit cubes a student gives to a peer which should lead to its reconstruction in the *Cubix Editor* virtual environment (Figure 1).

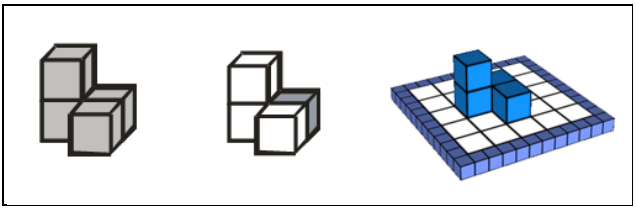

Figure 1: A cubical construction as a drawing and in the *Cubix Editor*

The experiment was carried out with 5th and 6th graders who had not started a systematic learning of mathematics. Thus the problems were propaedeutic with respect to the formation of mathematical notions, such as *ambiguity, redundancy* and *contradiction*. The same task was later proposed to pre-service and in-service teachers with the goal of bringing their attention to the necessity of enhancing the key competences related to communications in a mathematics context. More details about the experiment and the results could be found in [6].

2.3. THE ART OF POSING PROBLEMS BASED ON A SPECIFIC PICTURE OR A GRAPH

Let us consider some examples.

Example 1: A basket with eggs: *Ask questions about the situation on the picture* (Figure 2a).

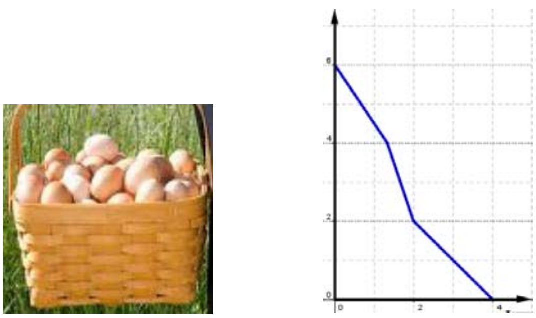

Figure 2: Verbalizing ideas based on a picture (a) or a graph (b)

Formulating questions that could be a basis for an interesting mathematic problem is a competence that is one of the aspects of communication. Finding appropriate connections with the real world and asking questions whose answer would be based on inquiries conveys the spirit of mathematics as it is used in real-life situations. It is such kind of competences that are enhanced by KeyCoMath, and especially in the context of the Mascil project whose main goal is to integrate the inquiry based learning (IBL) with the world of work and show how to implement this style in day-to-day teaching [7].

Example 2: A graph as a generator of stories - *Think of a story illustrated by the graph* (Figure 2b) [8]. The experiment with math teachers within a PD course on the inquiry based learning proved the efficacy of the problem. When solving it the teachers discussed various competences that could be gained and developed in its context – reading and interpreting a graph, generalizing and specifying, making abstractions.

2.4. CODING AND DECODING

Here is an example of a problem on decoding a text given in a dynamic file (Figure.3).

Problem: *Decode the words on the screen:*

Figure 3: A problem on decoding
http://www.math.bas.bg/omi/cabinet/content/bg/html/d23090.html
http://www.math.bas.bg/omi/cabinet/content/bg/ggb/d23090.ggb

The solution is based on geometric transformations. Various means could be used to facilitate the solution – a mirror, a transparent paper, etc. since the text could be considered as a set of points, then – as symbols and at the end – as words. Using a dynamic construction (although it is still not a typical tool in the Bulgarian schools) could be an introduction to specific dynamic software with all its facilities for explorations, thus enhancing the digital competence of the students.

2.5. THE 30 PROBLEMS IN 30 LANGUAGES CONTEST

This contest was designed and launched by Ivaylo Kortezov [9]. Each problem (as the title says) is in a different European language with 5 choices for the answer of which exactly one is correct, e.g.

Problem: Mitu täisarvu *n* rahuldavad võrratusi $2006 < 200+6n< 6002$?

A) 665 B) 666 C) 667 D) 668 E) 669.

The contest is a team one and a bonus is given for discovering the language of the formulation.

The next step is using various resources for solving a problem the way we do it in real-life situations. This was the idea behind the design of the *Mathematics with a computer* contest by a team of the Union of the Bulgarian Mathematicians and the IMI-BAS, being launched within a joint project with *Viva Cognita* (with *Vivacom* as operator) [10].

2.6. THE *MATHEMATICS WITH A COMPUTER* CONTEST

This is an online contest - every participant is free to choose where and how to enter the site within a fixed hour. The work conditions imitate the problem solving in the real world □ the participants could use Internet, software systems, books, advice of a friend, etc. Furthermore, they are provided with dynamic files which could facilitate the exploration and the solving of the problem (Figure 4).

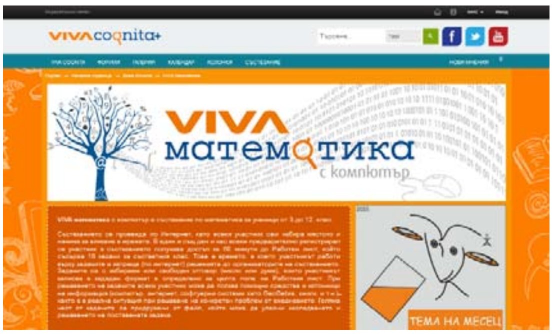

Figure 4: The *Mathematics with a computer* contest

You could find more about the contest in [10] and at http://vivacognita.org/.

Another contest in the same spirit is “The theme of the month” (Figure 5) which emerged as a means for preparation for the *Mathematics with a computer* but has confirmed its existence as an independent form. Its specific is the duration of the work – up to one month, which brings the spirit of problem solving closer to the genuine math problems. Such an approach is in harmony with the constructionism approach to mathematics expressed by Conrad Wolfram in a TED’s talk as follows [11]: *…correctly using computers is the silver bullet for making math education work… We want the students to be able to play with the math, interact with it, feel it. We want people who can feel the math instinctively. That's what computers allow us to do…*

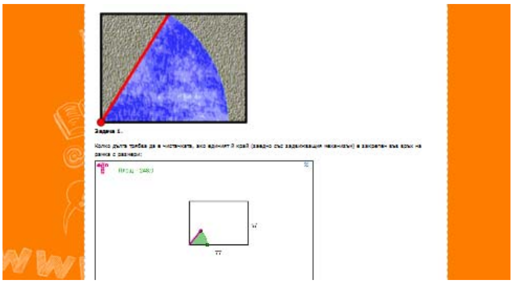

Figure 5: A fragment of the *Theme of September*

A virtual lab for school mathematics (VirMathLab) being developed at the IMI-BAS turns out to be especially suitable for preparation for these contests [12].

2.7. VIRMATHLAB – A VIRTUAL LAB FOR SCHOOL MATHEMATICS

The resources in the VirMathLab (Figure 6) are dynamic scenarios which could be used independently by students at different age and teachers as well as in a class setting. Their design and complementary materials are in support of the IBL and stimulate the development of a *confident, critical and reflective attitude towards ICT* – aspects of the digital competences identified as crucial by KeyCoMath.

Figure 6: The VirMathLab site
http://www.math.bas.bg/omi/cabinet/

The VirMathLab resources are based on the *GeoGebra* dynamic geometry software. The choice is justified by its being convenient to use, providing a good platform for making connections between different representations of mathematical objects, as well as various language versions, something of specific importance when it comes to communication of ideas in mother tongue and in foreign languages alike (Figure 7).

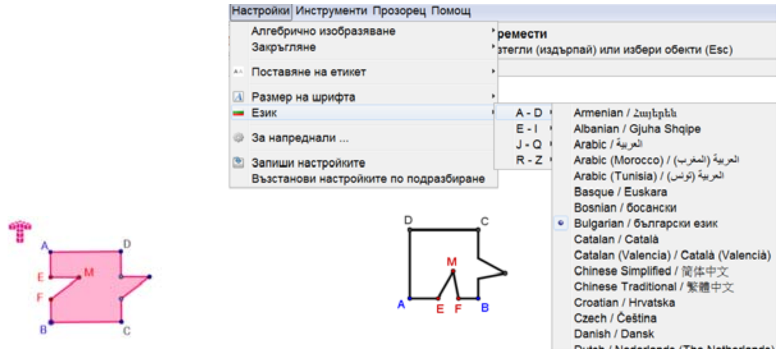

Figure 7: Changing the language in the *Geogebra* dynamic geometry software
http://www.math.bas.bg/omi/cabinet/content/bg/html/d18550.html
www.math.bas.bg/omi/cabinet/content/bg/ggb/d18553.ggb

2.8. Developing students' competences in doing research and presenting it – the RSI and the HSSI programs

To communicate verbally the meaning and the relevance of mathematics research to an audience larger than a few colleagues working in the same field is a competence which is difficult to acquire. How to present research projects in math and science to specialists in the corresponding field as well as to peers who are working on a large spectrum of science topics, is part of the tutors' duties within the Research Science Institute (RSI) [13]. This six-week summer program was launched more than 30 years ago by Admiral Rickover and Joann DiGennaro and is sponsored jointly by the Center for Excellence in Education, Virginia, and Massachusetts Institute of Technology (MIT). It is usually attended by 80 high school students, aged between 15 and 17, from about 20 countries from all over the world. The students attend lectures in science, philosophy, ethics, and humanities delivered by eminent researchers including Nobel Prize winners. The internships following the first week classes comprise the main component of the summer school. At the end of the program the students present a paper summarizing their results and give an oral presentation in front of a large audience at the RSI Symposium. Great importance is given to the need to capture the imagination of the audience while communicating mathematical concepts in a clear and understandable way. An example of a Bulgarian student who succeeded in doing this in the context of non-commutative algebras, dealt with putting a sock over the shoe during the oral presentation to demonstrate the notion of non-commutativity in an attractive way. The tutors direct their students to the general goal via a path traced by milestones (intermediate objectives) for enhancing their communication competence, e.g.:

- Presenting a mini-project using the same sample as the one for the final paper;

- Gradual filling of the sample starting with the background of the project, the methods used, considering partial cases and possible generalizations; classifying the cases of failure, etc.;
- Presenting the introductory part of their project in 3 minutes at a “posterless” session (with no props);
- Presenting the project in 5 minutes with any visual support they find appropriate.

Working in such an inquiry based style naturally develops competences important not only in the life of professional scientists but also in those of citizens of the knowledge/creativity based society – *planning, searching for and selecting appropriate information, integrating knowledge from different fields, and working in a team*. By passing along the milestones, the students build up a set of competences interwoven with predetermined objectives preparing them for their dynamic participation in society.

The good news from a national perspective is that a similar Institute (HSSI) for high school students (which focuses on mathematics, informatics and IT education) was launched in Bulgaria in 2000 and is functioning within the framework of the Institute of Mathematics and Informatics and the Union of the Bulgarian Mathematicians. The Bulgarian participants in RSI are passing the torch to the younger scientists-to-be with the sense of a mission, thus developing yet another competence, that of mentoring – possibly one of the most challenging form of communicating ideas.

3. CONCLUSION

Our long term experience in working with students, pre-service and in-services teachers alike shows that concentrating on the development of key competences in the context of mathematics, informatics and IT education can enrich existing educational strategies, methods and forms with properly designed learning environments for expressing an enhancing the communication competences.

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AN APPROACH TO ELIMINATION OF “EXPERIMENTAL-THEORETICAL GAP” IN TEACHING MATHEMATICS WITH DGS: EVALUATION OF EFFECTIVENESS

Maria Shabanova[1], Alexander Yastrebov[2], Tatiana Shirikova[3]

Abstract. Dynamical geometry software (DGS) is used in mathematics education since 1980s. DGS allows us to create dynamic drawings and experiment with them to obtain and verify hypotheses concerning properties of mathematical objects. Experimental way of learning is very simple for students, and dynamical visualizations have a high level of credibility. All this contributes to a widely held belief that deductive reasoning is no longer needed. In scientific literature, this phenomenon is known as “experimental-theoretical gap”. The name emphasize that one of the dualistic properties of mathematics is violated. The article presents the results of experimental and theoretical research. The purpose was to find a pedagogical technology of using DGS for composing mathematical propositions that would eliminate the risk of such a gap. Authors’ technology consists of three steps. At the first step students learn how to make right conclusions from computer experiments and how to take into consideration the limited capabilities of these experiments in verifying propositions. At the second step students learn how to use theoretical methods to verify correctness of experiments and adequacy of dynamical drawings. At the third step students learn how to use computer experiments to develop ideas of using theoretical methods to prove propositions.

1. INTRODUCTION

Dynamical geometry software (DGS) represent a separate class of software for research and educational purposes. The first program of this class (Cabri-g□om□tre) was created in 1985. At present, there exist more than fifty programs of such a kind. Only four programs of this class are used in Russia. They are The Geometer’s Sketchpad (1989), GeoNext (1999), GeoGebra (2002), 1C: Mathematical Constructor (2006). Dynamic geometry software has a lot of opportunities to support cognitive activity of students. DGS is used

- to construct a virtual model of an object under consideration;
- to explore its new properties;
- to verify hypotheses;
- to monitor the progress of reasoning and analytical calculations;

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- to develop an idea how to solve the problem.

DGS provides an ability to build teaching process using the methodology of experimental mathematics, but often these opportunities are used incorrectly. Facts are established experimentally and then are accepted without proof. Let us show it on an example of method applied for working with the following problem.

Problem 1. For a given circumference ω with center A and a point B within the circle, find locus of points X satisfying the following property: the distance between X and B is equal to the distance between X and ω.

Students constructed a locus without understanding the operation principles of the tool created by the teacher (figure 1). Their experiments verified that the locus is the ellipse with focuses at points A and B, but did not prove this fact (figure 2).

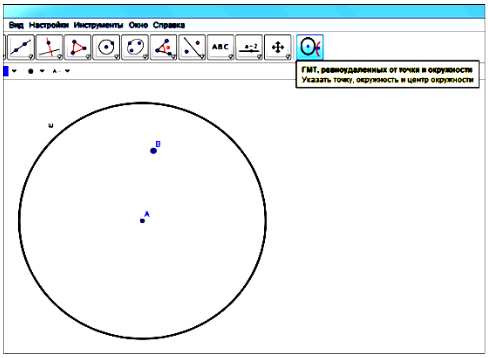

Figure 1

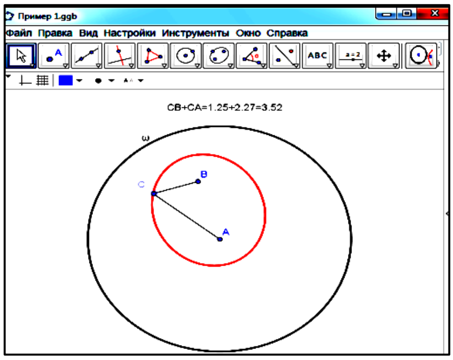

Figure 2

Practice show that systematical use of incorrect pedagogical scenarios generates a negative educational phenomenon named «experimental-theoretical gap» (J. Mason, 1991 [1]; G. Hanna, 2000 [2]; M. Mariotti, 2000 [3]). This name underlines the fact that application of DGS can sometimes disturb the balance and break natural connection between experimental and theoretical methods. The entire history of mathematics confirms the importance of these connections for cognition.

Let us list some negative effects:

- Dramatic decline in students’ (and even teachers’) motivation to use deductive method to ground facts obtained through computer experiments.
- Students’ unwillingness to use theoretical methods for solving problems and their preference for exploration computer experiments.
- General students’ preference for computer experiment, but not for theoretical statement of new problems.

The authors of present paper propose an approach allowing teachers to avoid experimental-theoretical gap.

2. THE MAIN POINTS OF AUTHORS' APPROACH TO ELIMINATION OF EXPERIMENTAL-THEORETICAL GAP

In order to solve research problems students should be able to find a rational combination of theoretical and experimental methods. We consider such ability as a final goal of education.

To reach this goal, we suggest that students should be guided through this skill step by step, mastering it at three levels.

Level 1. Mastering experimental methods and developing a critical attitude to experimental results.

Level 2. Mastering theoretical methods and using them for control and explanation of experimental results.

Level 3. Understanding relationships between theoretical and experimental methods of solving research problems.

These levels are put in practice through a system of so-called “advisable tasks” as well as through a special methodology of students' activity in solving each problem.

Generally speaking, our methodology is aimed to help students understand the experimental-theoretical dualism of mathematics.

The first level system include the following types of tasks:

- Make a computer experiment at the ready-made worksheets under given plan and analyze its results.
- Plan a computer experiment and select “the best” plan.
- Explain a construction of dynamical model for computer experiment and test it.
- Explain the reason of paradoxical results of computer experiment.
- Explain the reason of limited possibilities of computer experiment.

Let us consider a problem of the fourth type.

Problem 2. How many points are there on the graphic field (figure 3, figure 4, figure 5)? What are their coordinates?

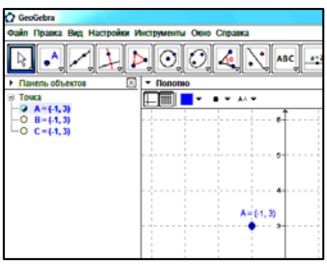

Figure 3.

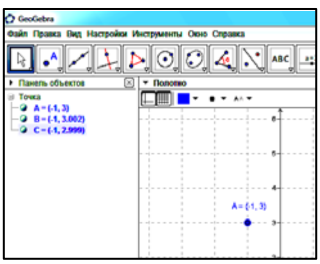

Figure 4.

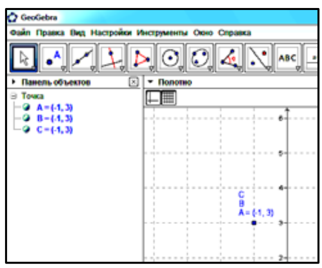

Figure 5.

We named this task “The Many Faces of a Point”. A teacher shows a graphic field of GeoGebra to a student and asks him three times, “How many points are there on the graphic field? What are their coordinates?”. In the first case, there is only one point. In the second case there are three points situated very closely to each other. The third case looks like the second one, but increase in accuracy does not change the coordinates of the points. The same construction was repeated three times. Problems of such a kind help create a belief that is very important for a world-view of a mathematician-experimentalist: any observation requires theoretical comprehension and interpretation.

On level 2 our system includes tasks of the following types.

- Construct a dynamical model of a figure, if you have no pre-image of that figure.
- Modify an algorithm of constructing the dynamical model according to theoretical basis selected for its construction.
- Describe and explain the algorithm of constructing the dynamical model.
- Explain the reason of coherent changes of the figure's properties.

Let us consider a problem of the last type.

Problem 3. Three points *M, N* and *K* are situated on the sides of a triangle in such a way that $AM = \alpha AB$; $BN = \beta BC$ and $CK = \gamma AC$. Find *experimentally* the relationship between the ratio of squares

$\frac{S_{MNK}}{S_{ABC}}$ and an expression $1-(\alpha+\beta+\gamma)+(\alpha\beta+\alpha\gamma+\beta\gamma)$.

Explain you result theoretically.

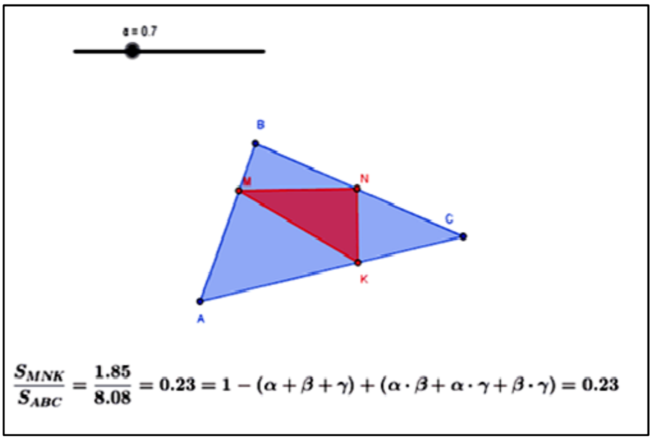

Figure 6.

The equality (figure 6) can be proved using the method of areas:

$$S_{MNK} = S_{ABC} - (S_{AMK} + S_{BNM} + S_{CKN}) .$$

On level 3 our system includes tasks of the following types:

- Reproduce the proof presented as a ready-made computer visualization.
- Prove a fact that was obtained experimentally.
- Use a computer experiment to find an idea of analytical solution of the problem.
- Use a computer experiment to control transformations and reasoning.
- Unite theoretical methods and computer experiments to formulate new research problems.

To demonstrate proposed technology let us come back to Problem 1. As it is a problem of the third level, we will use it though with some modifications.

Problem 4. For a given circumference ω with the center A and an arbitrary point B, construct a point X satisfying the following property: the distance between X and B is equal to the distance between X and ω. Find the locus of points X experimentally, and prove the result. Explore the locus for following cases: 1) B within the circle; 2) B outside the circle. Create GeoGebra's tool to construct these loci.

Point B can move. It can be inside or outside the circle. Students find a way to construct a point locus E for both cases by themselves and prove its correctness. Then students prove the hypothesis about the form of the locus. Ellipse in the first case (where $AE + EB = R = const$) and hyperbola in the second case (where $AE - EB = R = const$) (figures 7, 8).

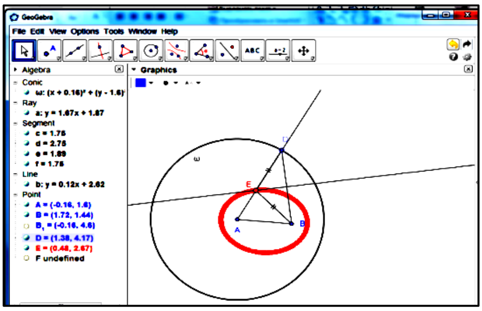
Figure 7.

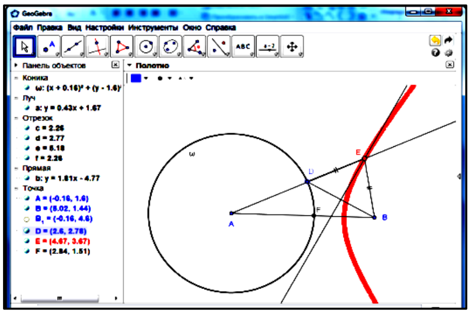
Figure 8.

3. Pedagogical experiment

A pedagogical experiment was carried out in 27 schools that were the pilot sites of the "Methods and information technologies in education" (MITE) project. It was conducted over 2010–2013 by both Russian and Bulgarian specialists. 676 middle school students as well as 42 teachers took part in it.

An Experimental Teaching Program is presented in the book for teacher. A Bank of Tasks and Instructions for Teachers were created for experimental work. The book includes 120 problems in total. We also used a basic textbook "Geometry 7–9" [4] and the electronic applications with workbooks [5], [6], [7].

The first level was reached when students were studied "First statements of geometry", a theme of geometry course (9 hr.). While studying this theme, students gradually accumulate knowledge about the simplest geometry figures and their properties. Later, a system of these statements plays the role of axioms for school geometry course.

The second level was reached when students were studied “Triangles” and subsequent themes of geometry course for the 7th grade (38 hr.). While studying these themes, students become familiar with postulates of constructive geometry and with basic tasks on constructing with a ruler and compass.

The third level starts with “Quadrilaterals” and goes on during study of all themes intended for 8th and 9th grades (138 hr in total). This theme include very many definitions and statements about properties and attributes of geometric figures. It provides a lot of space for experiments and logical deployment of theory.

4. METHODOLOGY OF DATA GATHERING

To collect the data we used longitudinal method. We carried out 15 measurements on the same sample of students. Measurements were carried out at the end of each theme. Each test included problems for testing skills of using a combination of theoretical and experimental methods. The results of problem solving were evaluated by two criteria: *K1* — success in solving diagnostic tasks by selected methods; *K2* — conformity of a method or a combination of methods with the demands of the task (to imagine, to convince, to explain, to prove). K1 was assessed using 50-point scale. Students could get from 0 to 50 points, depending on the degree of progress in solving a diagnostic task. The experimenters (teachers) actively participated in drafting of diagnostic tasks; therefore, there approaches to the assessment of students' solutions were pre-agreed. K2 criterion was assessed using 3-point scale: “0” - the selected method/methods does not comply with the requirement; “1” - partial compliance; “2” - full compliance.

Diagnostic tools are presented in [8]. Each test includes problems for testing skills of using a combination of theoretical and experimental methods.

We present an example of diagnostic task that pertains to the theme of “Quadrilaterals” (level 2).

Problem 5. On the dynamic model of the rhombus ABCD construct the bisector of the angle DCA. Find the angles of the rhombus, provided the bisector is perpendicular to the side AD. Explain the result.

Examples of students' solutions of the tasks and respective assessments made by the teachers are presented below.

Student A. If I approximate the angle *DKC* to 90^o, I will obtain the rhombus's angles of approximately 60^o and 120^o (figure 9). I checked it by constructing a rhombus with such angles. All is true (figure 10). (K1-40, K2-1).

Student B. Let us construct a rhombus in such a way that the bisector of the angle *DCA* is perpendicular to the side *DA*. We can start with construction of an equilateral triangle *ADC*. The angles of an equilateral triangle are 60^o each; therefore, the rhombus angles are 60^o and 120^o. (K1 -20, K2 -0).

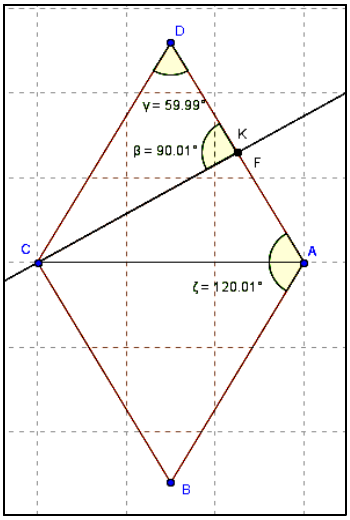

Figure 9.

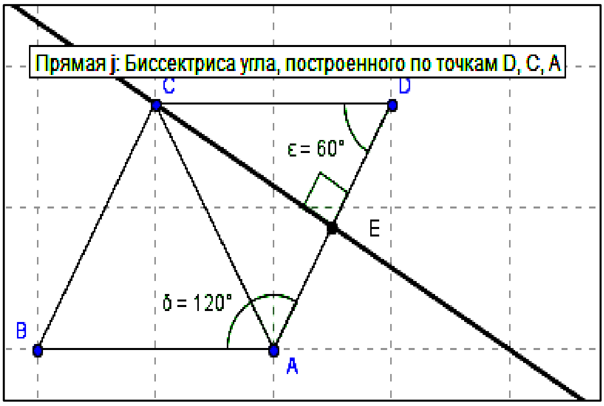

Figure 10.

Student C. By experimenting with the image of the rhombus, I have come to a conclusion that the bisector of the angle *DCA* is perpendicular to the side *AD*, if the angles *A* and *C* are equal to 120^o, and angles *B* and *D* are equal to 60^o. In order to explain our statements, we have to consider the triangle *CDA*. It is equilateral, because $AD = DC$ as the rhombus's side, and $DC = CA$, because the bisector *CK* is the height of this triangle. The diagonals of a rhombus are the bisectors of its angles, therefore, the angle $A = 2\measuredangle CAD$ A = 2. (K1-50, K2-2).

5. Data Analysis and Results

The measurements resulted in ranks of distributions, which are presented by diagrams 1 and 2. Diagram 1 shows the proportion of students who selected a method or combination of methods in full conformity with the situation. Diagram 2 shows the average score of success in diagnostic task solving.

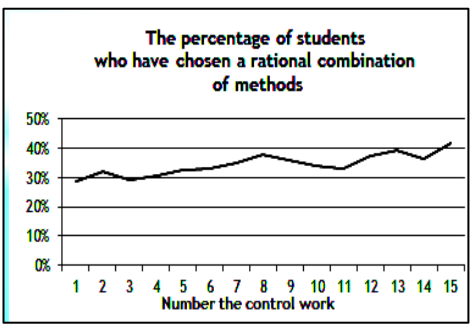

Diagram 1.

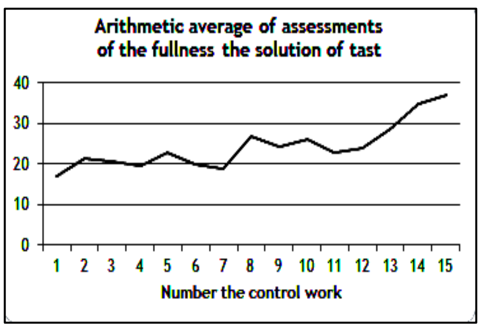

Diagram 2.

Analysis of these diagrams allows formulating the two pairs of hypotheses:

Type of hypothesis	Number of criterion	
	K_1	K_2
H_0	There are only random differences between distributions of rationality of method choice, obtained for solution of different diagnostic tasks.	There are only random differences between distributions of success in solving different diagnostic tasks.
H_1	There are non-random differences between distributions of rationality of method choice, obtained for solution of different diagnostic tasks.	There are non-random differences between distributions of success in solving of different diagnostic tasks.

We used Friedman's χ^2 criterion to verify the hypotheses. This criterion establishes only the statistical significance of differences between n distributions.

In this regard, we established the character of trends only by diagrams.

As a result of the calculations we have established that $\chi_r^2 = 149{,}3 > 124{,}3$ for K_1 (at significance level of 0.05 [9]), and $\chi_r^2 = 168{,}8 > 124{,}3$ for K_2. Therefore, in both cases we should reject hypothesis H_0 and accept hypothesis H_1.

The results of pedagogical experiment showed that proposed approach contributes to formation of skills of using rational combination of theoretical and experimental methods for solving research problems. However, this process is very slow and requires constant attention and support.

6. FINAL REMARKS

There are other approaches to solving the problem of preventing “experimental-theoretical gap”.

The most radical of them is to ban the use of DGS in teaching mathematics, until theoretical style of mathematical thinking of students is formed. Similarly, the use of calculators is banned over the period when students' numeracy is to be developed. However, practice shows that this way is not very effective in an open information space and due to availability of software.

Our approach is consistent with the proposals of other authors and integrates their pedagogical solutions. The approach involve the following goals:

- to introduce multilevel differentiated learning of logical reasoning and proof (V. Rigik, 2013 [10]);
- to review task material and include the tasks that require combining theoretical and experimental methods (R. Marrades, A. Gutierrez, 2000 [11]);
- to expand the range of semantic meanings and functions of proof and proving for students to master: *verification, explanation, systematization, discovery, communication, construction, exploration, incorporation* (G.Hanna, 2008 [12]).

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